BOUNDARY DEHN TWISTS ARE OFTEN COMMUTATORS

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BOUNDARY DEHN TWISTS

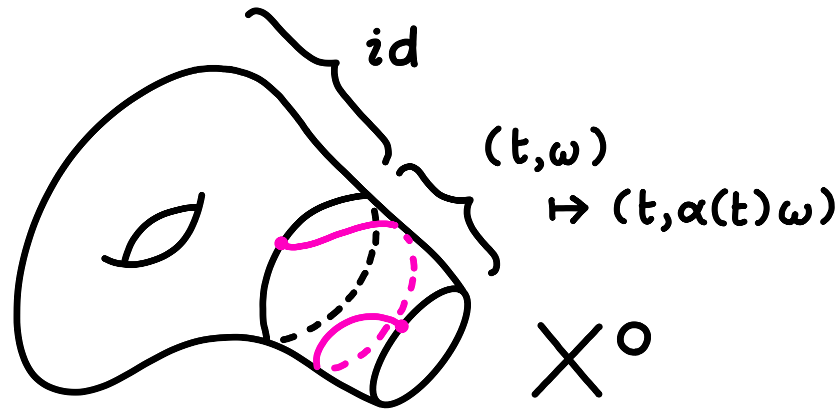
X^4 closed, smooth, $\pi_1(X) = 1$

$X^\circ := X \setminus B^4$, $\nu(\partial X^\circ) \cong I \times S^3$

$[\alpha]$ generator of $\pi_1(SO(4))$

$\delta_X : X^\circ \circlearrowleft$, $\delta|_{X^\circ \setminus \nu(\partial X^\circ)} = id$,

$\delta_X|_{\nu(\partial X^\circ)} : (t, \omega) \mapsto (t, \alpha(t)\omega)$



TOP isotopic to id rel ∂ (Orson-Powell '23)

SM not iso. to id rel ∂ for $X = K3$ (Baraglia-Konno '22, Kronheimer-Mrowka '20), $K3 \# S^2 \times S^2$ (J. Lin '20),

$K3 \# K3$ (Tilton '25), many complete intersections & $E(n)_{p,q}$ (Baraglia-Konno '26)

$\Rightarrow$ δ_X is an EXOTIC DIFFEOMORPHISM for such X

Q: "Where does this exoticity live"?

$\cdot^{ab} : \pi_0 \mathrm{Diff}(X^\circ, \partial) \xrightarrow{\text{abelianization}} (\pi_0 \mathrm{Diff}(X^\circ, \partial))^{ab} \cong H_1 B\mathrm{Diff}(X^\circ, \partial)$

Specific Q: When is δ_X^{ab} trivial?   Y. Lin '25 (using Global Torelli + work of Baraglia-Konno): δ_{K3}^{ab} is trivial

... ARE OFTEN COMMUTATORS

L. '26 : Write $K3 := \{ [z] \in \mathbb{CP}^3 \mid p(z) := z_0^4 + z_1^4 + z_2^4 + z_2 z_3^3 = 0 \}$

$[z] \mapsto [-z_0 : z_1 : z_2 : z_3]$, $[z] \mapsto [\bar{z}]$ restrict to commuting $a, c : K3 \circlearrowleft$

IFT $\Rightarrow$ Near $\mathfrak{z} := [0:0:0:1] \in K3$, a, c act as $A := \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$, $C := \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$

For any $R_a, R_c : [0,1] \to SO(4)$ from I_4 to A, C, $[R_a, R_c]$ gens. $\pi_1 SO(4)$

For $K3^\circ := K3 \setminus \nu(\mathfrak{z})$, write $a^\circ, c^\circ : K3^\circ \circlearrowleft$, $\begin{matrix} a^\circ|_{K3^\circ \setminus \nu(\partial K3^\circ)} := a & a^\circ|_{\nu(\partial K3^\circ)} : (t, \omega) \mapsto (t, R_a(t)\omega), \\ c^\circ|_{K3^\circ \setminus \nu(\partial K3^\circ)} := c & c^\circ|_{\nu(\partial K3^\circ)} : (t, \omega) \mapsto (t, R_c(t)\omega), \end{matrix}$

$\Rightarrow [a^\circ, c^\circ] = \delta_{K3} \Rightarrow \delta_{K3}^{ab}$ is trivial!

Not at all special to $X = K3$: Underlying this is a general construction we combine w/ an alg. geo. argument to write δ_X as a commutator ($\Rightarrow \delta_X^{ab} = 0$) for X any complete intersection or # thereof (Incl. K3, $K3 \# (S^2 \times S^2)$, $K3 \# K3$, the ∞ many cplete ints. BK showed have nontrivial δ_X) and more (Incl. $E(n)$)

$\Rightarrow$ **δ_X IS OFTEN A COMMUTATOR**, and thus δ_X^{ab} is often trivial, even for many X s.t. δ_X is known to be nontrivial!